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A mathematical model for the diffusion of emergency warning messages during CBRNe emergencies

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Abstract

Understanding the diffusion of warning messages is essential to adequately respond to emergency events and situations. This is especially true in urgent scenarios, that is situations where external events are happening at the same rate or faster than the diffusion process itself. In this paper, an information diffusion model (Bass model) is proposed to study the spread of warning messages during emergencies involving urgent diffusion dynamics, for example a CBRNe event. In the present study, the Bass model is applied to two hazardous materials transportation accidents reported in the literature: the Pittsburgh phosphorus oxychloride release and the precautionary evacuation occurred in Confluence due to toxic chemicals released after a train derailment. Warning data collected from the two accidents and reported in published literature studies were used in this work and fitted with the Bass model. The diffusion of emergency warning messages is modelled as a two-component system, where the spread of information is characterized by (a) a "broadcast process" that disseminates the emergency warning vertically (in the sense that many people are alerted simultaneously) and (b) a horizontal "contagion process" whereby people first hear of the event and then sequentially tell others (social media, word-of-mouth and peer-to-peer communication). The Bass model provided an excellent fit of the warning diffusion times related to both accidents suggesting that the very first phase of the warning process is sustained by a "broadcast" information diffusion process. However, after less than 1 hr from the beginning of the warning process the efficacy of its diffusion is dominated by the "contagion" component, that is the effectiveness of a robust social network between individuals. In conclusion, the Bass model proved to be a handy tool to assess epidemics spreading of information from the people who adopted the information. Our results suggest that the general Bass model applied to diffusion of emergency warning has the potential to provide key information in the management of emergencies. This approach can be applied right away by professional communicators, advisors and decision-makers in case of a CBRNe event.

KEYWORDS
Bass model, CBRNe, emergency, spread of information

1 | INTRODUCTION

Chemical, biological, radiological, nuclear and explosive (CBRNe) terrorism is a form of terrorism involving the use of weapons of mass destruction. According to the University of Maryland's Global Terrorism Database (GTD & Global Terrorism Database, 2020), from 1970 to 2014 CBRNe weapons have been extensively used across the world for a total of 143 attacks, of which 35 biological, 95 chemical and 13 radiological. Therefore, there is growing concern among homeland security professionals, that terrorist groups will release CBRNe materials in an attack against civilian populations. In fact, following 11 September 2001 and the anthrax attacks in America of the following month, the international community came to believe that the reality of a CBRNe event has to be accepted. Furthermore, recent developments in the Middle East have revitalized awareness of the threat of attacks involving CBRNe weapons. Therefore, it is generally accepted that there is a realistic possibility of some form of unconventional terrorist attack in the western world and that this could involve CBRNe material (D'Arienzo, Pinto, Sandri, & Zagarella, 2017). The authorities need to consider how the community is likely to react, and collaborative efforts are needed to identify what steps would be required to mitigate the effects of such an event. Information management is a key element in an emergency scenario, such as a CBRNe attack. In a crisis or disaster situation, information management ensures the accuracy of the information that managers rely on to make critical decisions. For communicators, it is vital to know that every emergency, disaster or crisis evolves in phases and that the communication must evolve in tandem. During a crisis, unpredictable and unusual events, or unstable and dangerous situations may bring about abrupt change. Understanding the pattern of a crisis can help communicators anticipate the information needs of the public, stakeholders and the media. In particular, emergency warning strategies to alert the public to potential danger in areas surrounding hazardous facilities are of critical importance.

The diffusion of emergency warnings resembles diffusion of other types of information or communications, except that it occurs in a shorter time period, and the consequences of not receiving the massage are usually more severe. The basic mathematical function is a logistic function. The cumulative proportion of people receiving the warning forms an S-curve, which is determined by the exponential form of the initial alerting process and the logistic form of the subsequent contagion of the warning and message through the population.

According to recent research (Martin-Shields, 2019; Rogers & Sorensen, 1988; Vihalemm, Kiisel, & Harro-Loit, 2011; Warren, 2015), the diffusion of warning messages is characterized by two main components: vertical and horizontal sources of information. Broadcast media (such as radio and television), together with governments, politics and news reports, are vertical systems since the information is single-source, broadcasts down and diffuses out. As a consequence, the emergency warning is disseminated in a centralized way, in the sense that many people are alerted simultaneously. On the other hand, horizontal media include cellular telephones, crowd sourcing platforms and social media. This type of media provides a tool to share and generate information using a peer-to-peer network system. This process if often referred to as contagion component, since people first hear of the event and then sequentially tell others.

With this in mind, this paper seeks to address the following questions: do people generally use horizontal or vertical media to gather information and to take action during an emergency? What is the time evolution of the two components over time? In the present study, an information diffusion model (namely the Bass model) is introduced to assess the spread of warning messages during emergencies involving urgent diffusion dynamics. The mathematical model for the diffusion of emergency warning messages was then applied to two hazardous materials transportation accidents in the United States: (a) the Pittsburgh phosphorus oxychloride release and (b) the precautionary evacuation occurred in Confluence following a train derailment.

2 | RELEVANT LITERATURE

Communication is an essential component of disaster management, and a growing body of literature has examined the impact of a prompt dissemination of warning messages during emergencies. Unavoidably, the research summarized in this section is only a subset of a large body of work done on emergency alerts and warnings.

Channels of delivery of warnings and alerts during emergencies can be viewed by the public as official (Cutter, 1987; Perry & Green, 1982; Saarinen & Sell, 1985), credible (Perry, 1987; Stallings, 1984), familiar (Lindell & Perry, 1987; Perry & Green, 1982; Perry & Lindell, 1986) or may involve human interaction (Cutter, 1987; Gray, 1981; Perry, Lindell, & Greene, 1981). Each of these channels is effective in some settings, but not all. As a general rule, how a recipient receives a warning message may influence an individual's perception on the risk and threat and therefore affects the time required to take a protective action. As an example, social media, phone applications and online messaging among friends and family are likely to play a key role during an emergency. According to a recent study by Bagrow and colleagues (Bagrow, Wang, & Barabási, 2011), word spreads fast and far during an emergency. In their study, the authors found that large-scale emergencies, such as bombings and plane crashes, trigger a sharp spike in the number of phone calls and text messages (SMS) sent by eyewitnesses in the vicinity of the disaster. The effectiveness of SMS and other text messages as a tool for disaster warning is confirmed by other studies (Bean et al., 2016; Egnoto, Svetieva, Vishwanath, & Ortega, 2013; Eriksson, 2010; Nugraheni & Vries, 2015).

Along the same lines, many studies have been published on the use of social media as complementary channels for emergency alerts (van Dijl, Zebel, & Gutteling, 2018; Eriksson & Olsson, 2016; Helsloot & Groenendaal, 2013). As a matter of fact, social media usage is one of the most popular online activities. At present, about 3 billion people use social media worldwide, meaning that social media are used by one-in-three people in the world (The rise of social media, 2020). For this reason, social media platforms are widely used during disaster events by emergency responders, people in the affected community and global onlookers (Hughes & Palen, 2009) who converge there to seek and share information (Brynielsson et al., 2017; Cheng, 2016; van Dijl et al., 2018; Kim, Bae, & Hastak, 2018; Wukich, 2016). However, there are also many challenges related to the use of social media in the crisis context, for example the spread of misinformation on these platforms (Department of Homeland Security, 2018; Truong, Caragea, Squicciarini, & Tapia, 2014; Wang & Zhuang, 2018).

Against this backdrop, in recent years there has been considerable interest in modelling the dynamics of information spreading during disasters and emergencies (Brynielsson et al., 2017; D'Agostino, D'Antonio, De Nicola, & Tucci, 2015; Goyal, Bonchi, & Lakshmanan, 2010; Pastor-Satorras, Castellano, Van Mieghem, & Vespignani, 2015). In the present study, the Bass model is introduced to evaluate the spread of warning messages during emergencies. The original Bass model, popular in the field of marketing, was originally developed for understanding the diffusion of innovations and consumer durables (Bass, 1969). According to this model, the diffusion process can be described as the sum of a logistic growth and an exponential growth (see section 3.2). The diffusion of emergency warnings and alert resembles diffusion of other types of information or communication. Similarly to the Bass model, the cumulative number of people receiving warning forms an S-curve determined by the exponential trend of the initial alerting process and the logistic form of the subsequent phase (Rogers & Sorensen, 1988). For this reason, the Bass model can be more generally applied to the diffusion of information. In 2009, Hsiao (Hsiao, Jaw, & Huan, 2009) applied the Bass model to assess how information diffusion influences tourists' consumption patterns. Interestingly, in another work, Rand and colleagues (Rand, Herrmann, Schein, & Vodopivec, 2015) studied the diffusion of information comparing Twitter data with the Bass model and with the independent cascade model (Goldenberg, Libai, & Muller, 2001; Kempe, Kleinberg, & Tardos, 2015). The major difference between the present study and Rand et al. (2015) is that Rand implemented an agent-based Bass model (Rand & Rust, 2011) meaning that each individual (agent) has one of two states at each time step: (a) unaware or (b) aware. At the beginning of the simulation, all agents are unaware but over time each agent's state can vary probabilistically. Once an agent becomes aware, it remains aware for the rest of the simulation. In this study, the authors concluded that the models fit qualitatively similarly, but the diffusion patterns are quite different from each other.

The general approach presented in our study is similar to Rand et al. (2015) with the exception that our model is not agent-based. Another major difference is that in this study, we compared the Bass model with data from post-event surveys, while Rand (Rand et al., 2015) compared the above-mentioned models to data collected from social media (Twitter).

3 | MATERIAL AND METHODS

3.1 | A model for warning time diffusion

Previous research has found that risk area residents receive warnings from the official warning network of authorities and the news media (i.e. vertical systems) and also from an informal warning network of peers (i.e. horizontal system). In some instances, the informal warning network can account for a significant proportion of all first warnings (Lindell & Perry, 1992). This finding suggests that warnings can be modelled as a process comprising two components: (1) the official (broadcast) component and (2) the informal (contagion) component (Rogers & Sorensen, 1988, 1989). The general mathematical specification of the diffusion curve is reported in Rogers and Sorensen (1988) and is recalled here:

$$\frac{dN(t)}{dt} = k\left[a_1(N-n)\right] + (1-k)\left[a_2 n(N-n)\right] \tag{1}$$

where k denotes the proportion of people getting informed during a notification period, a_1 (broadcast component) denotes the alert notification parameter revealing the alert notice efficiency, and a_2 (contagion component) denotes the communication and diffusion parameter showing the efficiency of alert notice. N denotes the proportion of people who should receive the notice, and n denotes the proportion of people who have already received the notice at different period of time.

Unlike the official warning network, which is organized and planned in advance, the informal warning network emerges during the incident from proximity (neighbours), kin (relatives) and other social ties (friends and coworkers). It is thus possible for planners to identify the components of the formal warning network, estimate the cumulative distribution of warning reception times and modify the network design if the resulting warning reception distribution is inadequate.

3.2 | The two-component Bass model

The Bass diffusion model was originally developed by Frank Bass [39] and describes the process of how new products get adopted as an interaction between users and potential users. On a wider level, the Bass model can be applied more generally to the diffusion of information [42]. The original model is based on the assumption that people get their information from two sources: advertising and word-of-mouth. In his 1969 article (Bass, 1969), Bass suggested that the following differential equation can be used to represent the diffusion process:

$$\frac{dN(t)}{dt} = \underbrace{qN(t)\left(1-\frac{N(t)}{m}\right)}_{\text{Effect of imitators (logistic growth)}} + \overbrace{p\left(m-N(t)\right)}^{\text{Effect of innovators (exponential growth)}} \tag{2}$$

where in the original model, $N(t)$ is the cumulative number of adopters at time t, m is the total number of potential buyers of the new product, p is the advertising or innovation coefficient, and q is the imitation or word-of-mouth coefficient. It is worth noting the resemblance between Equation (1) and Equation (2), mathematically equivalent. Equation (2) shows that the Bass model can be regarded as a combination of the exponential model (for early adopters) and the logistic model (for imitators).

The coefficient of innovation p is so called because its contribution to new adoptions does not depend on the number of prior adoptions. Since these adoptions were due to some influence outside the social system, the parameter is also called the *parameter of external influence*. The coefficient of imitation q received its moniker because its effect is proportional to cumulative adoptions $N(t)$, implying that the number of adoptions at time t is proportional to the number of prior adopters. In other words, the more people talking about a product, the more other people in the social system will adopt. This parameter is also referred to as the *parameter of internal influence*.

By analogy with the original formulation of the Bass model, in this work the assumption is made that people get their information from two sources: broadcast (vertical media) and contagion (horizontal media). In essence, the Bass model may be used to describe the fractional change in a population's awareness of a piece of information by Equation (2). In our new formulation, $N(t)$ is the cumulative number of aware people at time t, m is the total number of potential people that can be reached by the information, p is the broadcast coefficient, and q is the contagion coefficient. Assuming $F(t)=N(t)/m$, where $F(t)$ is the aware fraction of the population as a function of time t, the Bass model formulated in Equation (2) can be restated as:

$$\frac{dF(t)}{dt}=\left[p+qF(t)\right]\left[1-F(t)\right] \tag{3}$$

or:

$$\frac{F(t)'}{\left[1-F(t)\right]}=\left[p+qF(t)\right] \tag{4}$$

where p is the broadcast coefficient, and q is the contagion coefficient. Traditionally, q is an order of magnitude greater than p, representing the fact that social communication has a greater effect on adoption decisions than broadcast effects. The equation can be interpreted as describing a hazard rate, that is, the conditional probability that a person will become aware of information at time t given that they are not yet aware. In this case, the hazard rate $F(t')/\left[1-F(t)\right]$ is the sum of a constant broadcast effect p and a contagion (or peer-to-peer) effect $qF(t)$ that scales linearly in the fraction of population aware.

If $N\left(t=t_0=0\right)=0$, simple integration of Equation (2) gives the following distribution function to represent the time-dependent aspect of the diffusion process. That is,

$$N(t)=m\left(\frac{1-e^{-(p+q)t}}{1-\frac{q}{p}e^{-(p+q)t}}\right) \tag{5}$$

or, in terms of probability distribution:

$$F(t)=m\left(\frac{1-e^{-(p+q)t}}{1-\frac{q}{p}e^{-(p+q)t}}\right) \tag{6}$$

Equation (5) yields the S-shaped diffusion curve captured by the Bass model. In fact, for this curve, the point of inflection (which is the maximum penetration rate, $\left[dN(t)/dt\right]_{max}$ occurs when:

$$N\left(t^*\right)=m\left(\frac{1}{2}-\frac{p}{2q}\right) \tag{7}$$

$$t^*=-\frac{1}{p+q}log\left(\frac{p}{q}\right) \tag{8}$$

$$\frac{dN\left(t^*\right)}{dt}=m\left(\frac{q}{4}+\frac{p}{2}+\frac{p^2}{4q}\right) \tag{9}$$

The variables in the Bass model that can be calculated from m, p, q and t are the following:

- $F(t)'$, the (non-cumulative) portion of m that is aware at time t. This is given by:

$$F(t)'=\frac{p(p+q)^2e^{-(p+q)t}}{\left[p+qe^{-(p+q)t}\right]^2} \tag{10}$$

- $F(t)$, the portion of m aware by time t. This is given by Equation (6).
- The number $N(t)$ of aware individuals, in turn, can be split into two contributions: (a) individuals that become aware due to outside effects (i.e. information from outside the network) with probability p. In the original Bass model, these are called "innovators," N_{inno} and are given by (see also Equation 2):

$$N_{inno}=p\left[m-N(t)\right] \tag{11}$$

(b) individuals that with probability q become aware due to the social network. In the original formulation, they are called "imitators," N_{imit} and are given by (see also Equation 2):

$$N_{imit}=(q/m)N(t)\left[m-N(t)\right] \tag{12}$$

Naturally, $N=N_{inno}+N_{imit}$.

If $q > p$, then contagion effects (i.e. the possible effect of a social network) dominate the broadcast component (i.e. the effect of vertical media) and the plot of $dN(t)/dt$ against time (t) will have an inverted U shape (bell-shape). By varying p and q, we can represent many different patterns of diffusion of information quite well.

In a real context (e.g. CBRNe emergency and consequent diffusion of information), the Bass model can be viewed as a discrete-time model in which each individual has one of two states at each time step t: (a) unaware or (b) aware. At the beginning of the process, all people are considered to be unaware. Over time, unaware people are likely to become aware. Their state changes with a probability that reflects broadcast and contagion effects.

To conclude, the probability that an individual becomes aware may depend on two circumstances: (a) *Broadcast*. With probability p an unaware individual becomes aware due to outside effects (i.e. information from outside the network) where p is the coefficient of innovation. (b) *Contagion*. With probability q, an unaware person becomes aware due to social networks and peer-to-peer communication.

3.3 | Modelling warning and response in two hazardous materials transportation accidents in the United States: Pittsburgh and Confluence case studies

In the present paper, the Bass model described in section 3.2 was applied to two hazardous materials transportation accidents in the United States: i) the Pittsburgh phosphorus oxychloride release and the ii) precautionary evacuation occurred in Confluence. The Bass model was fitted to data derived from post-event surveys reporting the effectiveness of the warning system in terms of the timing of warning receipt by the population. A detailed description of the two accidents along with the original survey data is reported in (Rogers & Sorensen, 1989).

3.3.1 | Pittsburgh phosphorus oxychloride release

On Saturday, 11 April 1987 a westbound Conrail freight train derailed in Pittsburgh (Pennsylvania), prompting police to evacuate hundreds of homes and to ward off aircraft above the accident. Four tank cars containing hazardous materials were derailed, and sparks resulting from the accident ignited a fire. One overturned tanker car leaked between 100 and 400 litres of phosphorus oxychloride. The chemical is a toxic, colourless liquid that gives off a pungent odour and can cause skin burns. However, none of the hazardous materials involved in the accident ignited. Because of the involvement of hazardous materials, Pittsburgh emergency personnel initiated an evacuation upon arrival at the scene about 20 min after the accident. Apparently recognizing signs of potential danger, some local residents in immediate adjacent areas had already begun to evacuate. Up to 22,000 people were evacuated as the initial evacuation area was expanded to accommodate changing weather conditions.

3.3.2 | Confluence evacuation

On Wednesday, 6 May 1987 at 4:10 a.m., a freight train derailed in Pennsylvania and smashed into a railroad control tower, killing one man and spewing chemical fumes that forced 1,000 people to evacuate an entire town. During the accident, 21 of 27 empty tank cars carrying product residues, including propane, chlorine, caustic soda, carbon disulphide, methyl chloride, chloroform and isobutane jumped the tracks. Because tank cars carrying residue can haul up to 3% of the load, the exact amount of products remaining in the cars was unknown. As a consequence, 986 people were evacuated from their homes for over 12 hr. A 3-min non-stop siren blast was sounded, which primarily alerted the volunteer firemen as residents could not be aware of the siren blast's specific meaning, although it could serve as an alert to those who heard it.

3.3.3 | Confluence evacuation

Warning in both accidents primarily consisted of route alerting (portable sirens and loudspeakers) and door-to-door warning. These warning methods account for the majority of the warnings received (59% and 89%, respectively) (Rogers & Sorensen, 1989). Furthermore, about 67% and 28% of respondents reported visible or audible signs of the disaster. Route alerting methods of warning took 60 to 90 min in Pittsburgh, while portable sirens averaged just over 30. On the other hand, in Confluence loudspeakers and door-to-door alerting took about an hour on average in Confluence (Rogers & Sorensen, 1989).

3.3.4 | Warning data collected from the two accidents

For both accidents specific surveys were conducted aimed at assessing the timing of warning receipt with respect to the occurrence of the accidents (Rogers & Sorensen, 1989). Approximately 9 weeks after the accident, two surveys of residents in the Bloomfield section of Pittsburgh were conducted, based on a mail-back questionnaires and telephone interviews. The response rates were 29.3% and 51%, respectively. Regarding the Confluence precautionary evacuation, telephone interviews were conducted from October 20 to 28 (1987), approximately 22 weeks after the accident. Telephone interviews resulted in an 89.8% response rate. For both accidents, data regarding the cumulative proportion of population warned by time of receipt in terms of minutes into the event are reported in the results section. The original measurement difficulties were evidenced by the proportion of respondents that reported receiving warning prior to the occurrence of the accidents (Rogers & Sorensen, 1989). Most likely, this was due to the way people think about and recall time.

4 | RESULTS

The Bass model detailed in section 3.2 was fitted to data from accidents described in sections 3.3.1 and 3.3.2. In particular, the presence of an innovation (p) and imitation (q) component was estimated relying on warning data collection from the two accidents (Rogers & Sorensen, 1989). Non-linear regression of data related to warning times during the two accidents was performed using a in-house Python-based code (https://www.python.org/). It is worth noticing that the original survey data regarding the timing of warning receipt (Rogers & Sorensen, 1989) begin at $t = -60$ min from the accident, that is representing people that received warning prior to the occurrence of the accidents. As stated by the authors, this inconsistency is related to difficulties by respondents in recalling the exact time of warning receipt. Without any loss of generality, in the present study data are rescaled by 60 min, indicating the occurrence of the accidents at $t = 0$ min.

Consistently with the Bass model and with the two-component diffusion model described by Equation (1), warning receipt data for both accidents are characterized by an S-shaped curve (Figures 1a and 3a), thereby suggesting the presence of a p (innovation) and q (imitation) component. Results related to the Pittsburgh phosphorus oxychloride release accident are shown in Figures 1 and 2. Figure 1a shows the cumulative number of warned individuals after the occurrence of the accidents. Literature data taken from Rogers and Sorensen (1989) are reported (dotted line + circles) and fitted with the Bass model. From non-linear regression, it resulted

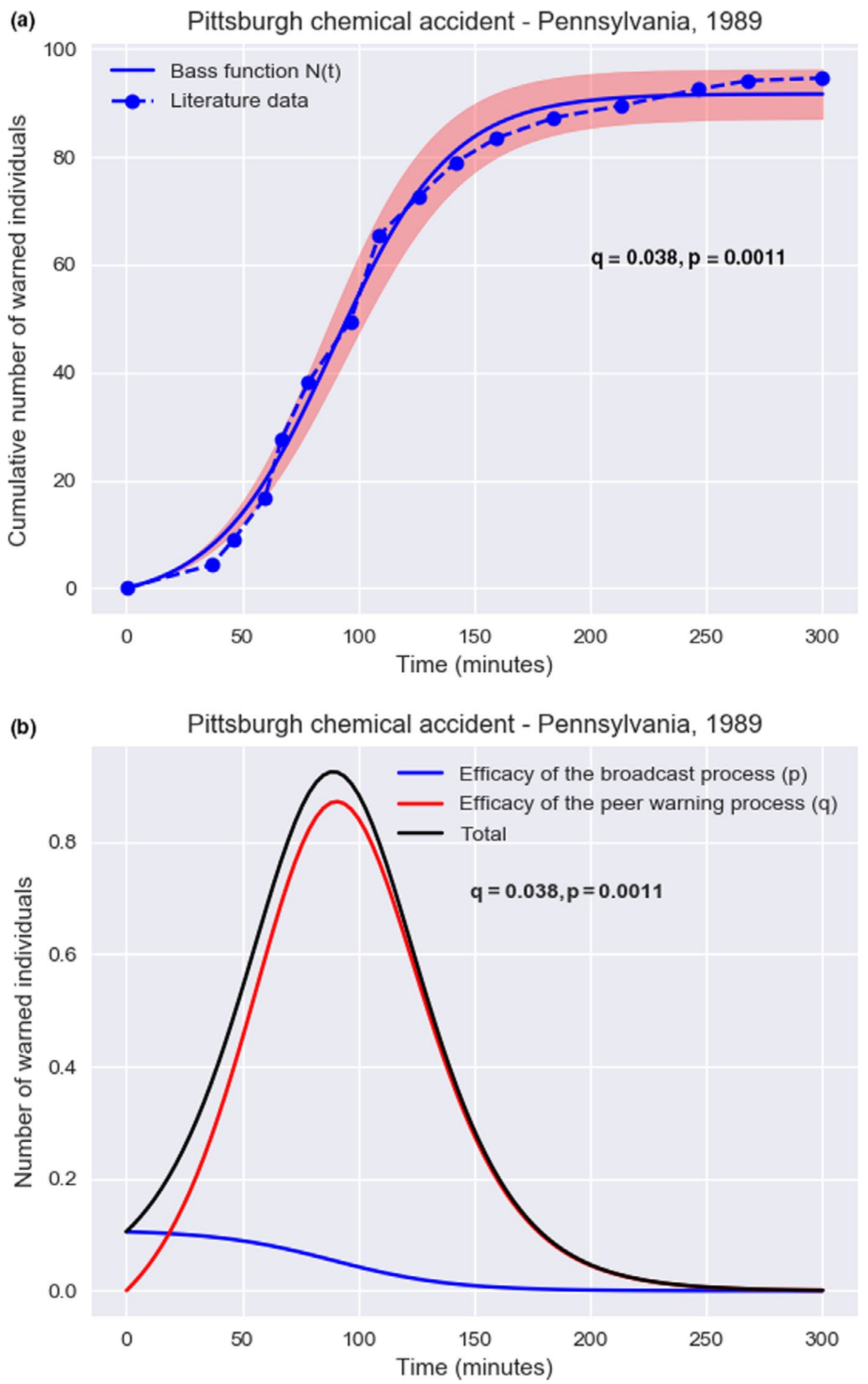


FIGURE 1 Application of the Bass model to the Pittsburgh phosphorus oxychloride release accident. (a) Literature data taken from Rogers and Sorensen (1989) are reported (dotted line + circles) and fitted with the Bass model. From non-linear regression, it resulted $q = 0.038$ and $p = .0011$. The red area represents 95% confidence interval in the fit. (b) The differential number of warned individual is represented, divided into three categories: Black line = total number of warned individuals. Blue line = Number of individuals warned by the broadcast process (related to q) and Red line = Number of individuals warned by the peer-to-peer process (related to p) [Colour figure can be viewed at wileyonlinelibrary.com]

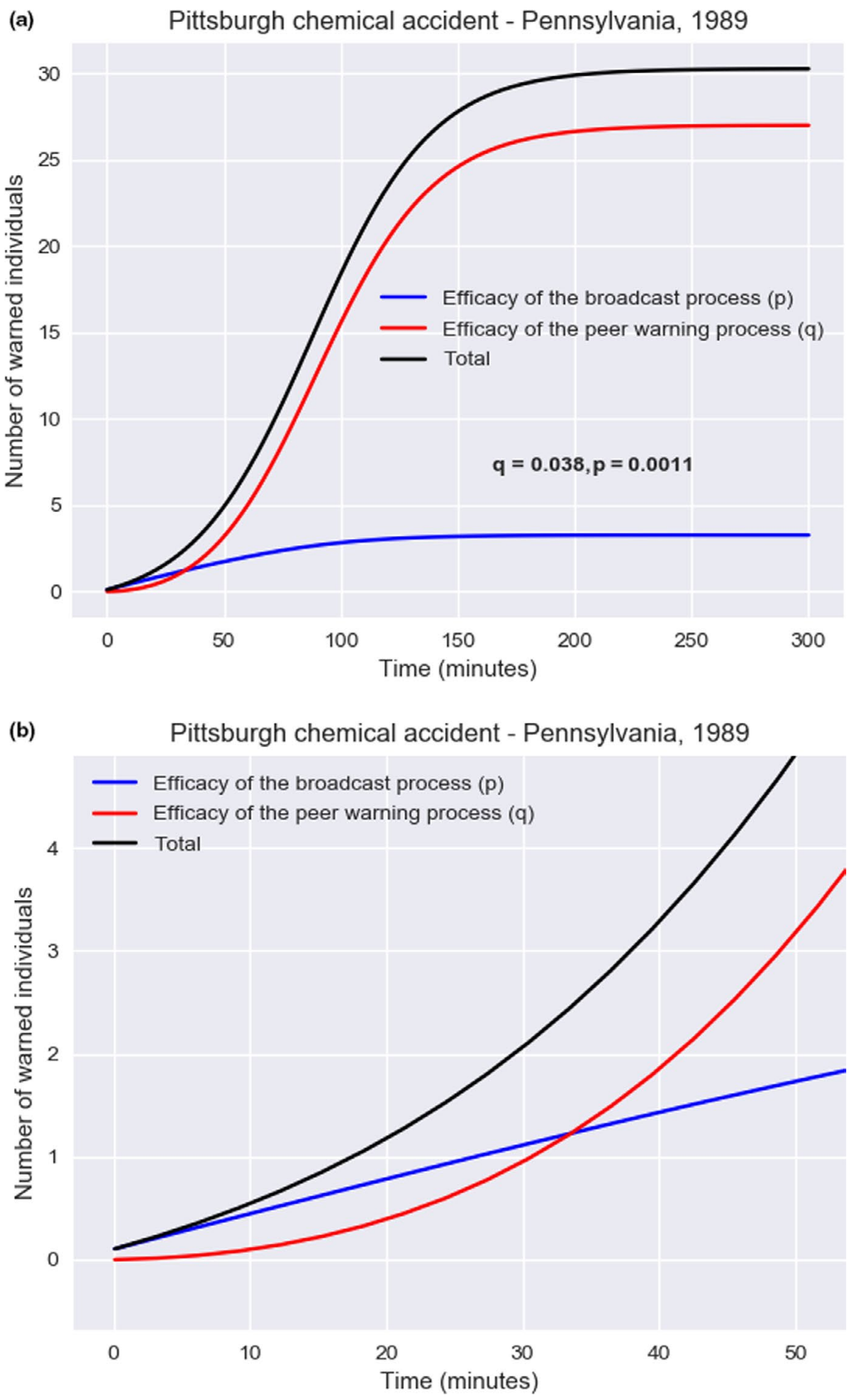
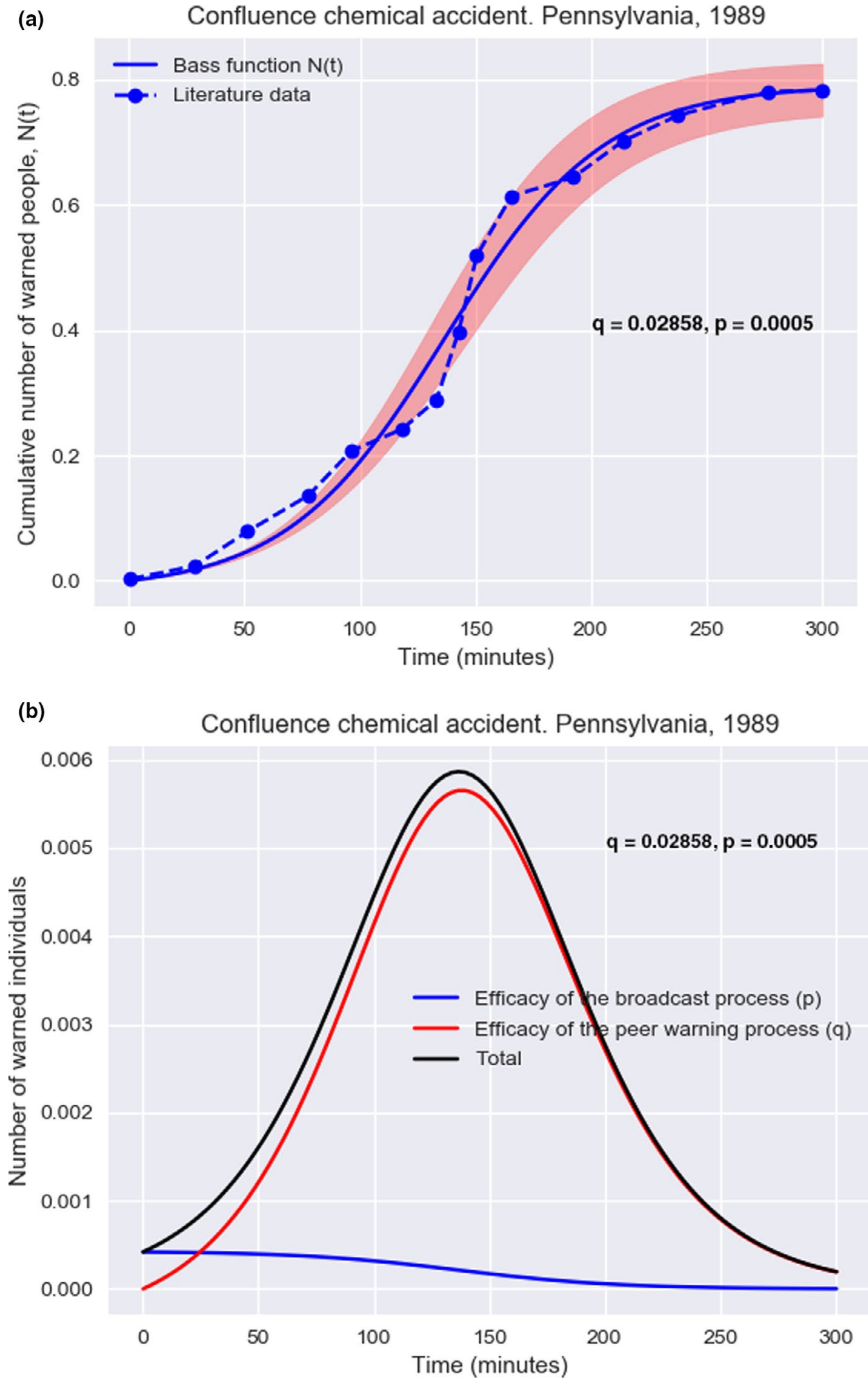


FIGURE 2 Application of the Bass model to the Pittsburgh phosphorus oxychloride release accident. (a) The cumulative number of warned individual is represented, divided into three categories: Black line = total number of warned individuals. Blue line = Number of individuals warned by the broadcast process (related to *q*) and Red line = Number of individuals warned by the peer-to-peer process (related to *p*). (b) Detail in the time slot 0–50 min [Colour figure can be viewed at wileyonlinelibrary.com]

$q = 0.038$ and $p = .0011$. Figure 1b shows the differential number of warned individual divided into three categories: total number of warned individuals, number of individuals warned by the broadcast process (related to *q*) and number of individuals warned by peer-to-peer process (related to *p*). The red area represents 95% confidence interval in the fit. Finally, Figure 2a,b shows the cumulative number of warned individual associated with p and q components.

Results related to the Confluence accident are presented in a similar way in Figures 3 and 4. Figure 3 shows the cumulative number of warned individuals after the accident. Literature data taken from Rogers and Sorensen (1989) are reported (dotted line + circles) and fitted with the Bass model. Non-linear regression analysis provided the following values for the imitation and innovation parameters, respectively: $q = 0.02858$ and $p = .0005$. Figure 4a,b illustrates the cumulative number of warned individual associated with p and q components.

FIGURE 3 Application of the Bass model to the Confluence precautionary evacuation. (a) Literature data taken from Rogers and Sorensen (1989) are reported (dotted line + circles) and fitted with the Bass model. From non-linear regression, it resulted $q = 0.02858$ and $p = .0005$. The red area represents 95% confidence interval in the fit. (b) The differential number of warned individual is represented, divided into three categories: Black line = total number of warned individuals. Blue line = Number of individuals warned by the broadcast process (related to q) and Red line = Number of individuals warned by the peer-to-peer process (related to p) [Colour figure can be viewed at wileyonlinelibrary.com]



5 | DISCUSSIONS

Understanding the effectiveness of warning messages during an emergency scenario is of critical importance. In fact, an effective and timely warning system has the potential to bring about the rapid implementation of protective measures thereby minimizing casualties and reducing damages. In the Internet world, people communicate with each other and form various virtual communities based on social networks, which lead to a complex and fast information spread pattern of emergency events (Cheng, 2016; Wukich, 2016). A large and growing body of literature has investigated the epidemic diffusion of information in complex networks and a number of models have been proposed (Brynielsson et al., 2017; D'Agostino et al., 2015; Goyal et al., 2010; Pastor-Satorras et al., 2015).

There has been considerable previous work understanding and modelling the diffusion of information during emergencies. Recent research suggested that the diffusion of warning messages can be modelled as a combination of a formal (vertical information) and an informal (horizontal) component (Rogers & Sorensen, 1991; Lindell & Perry, 1987; Lindell, Prater, & Peacock, 2007; Rogers &

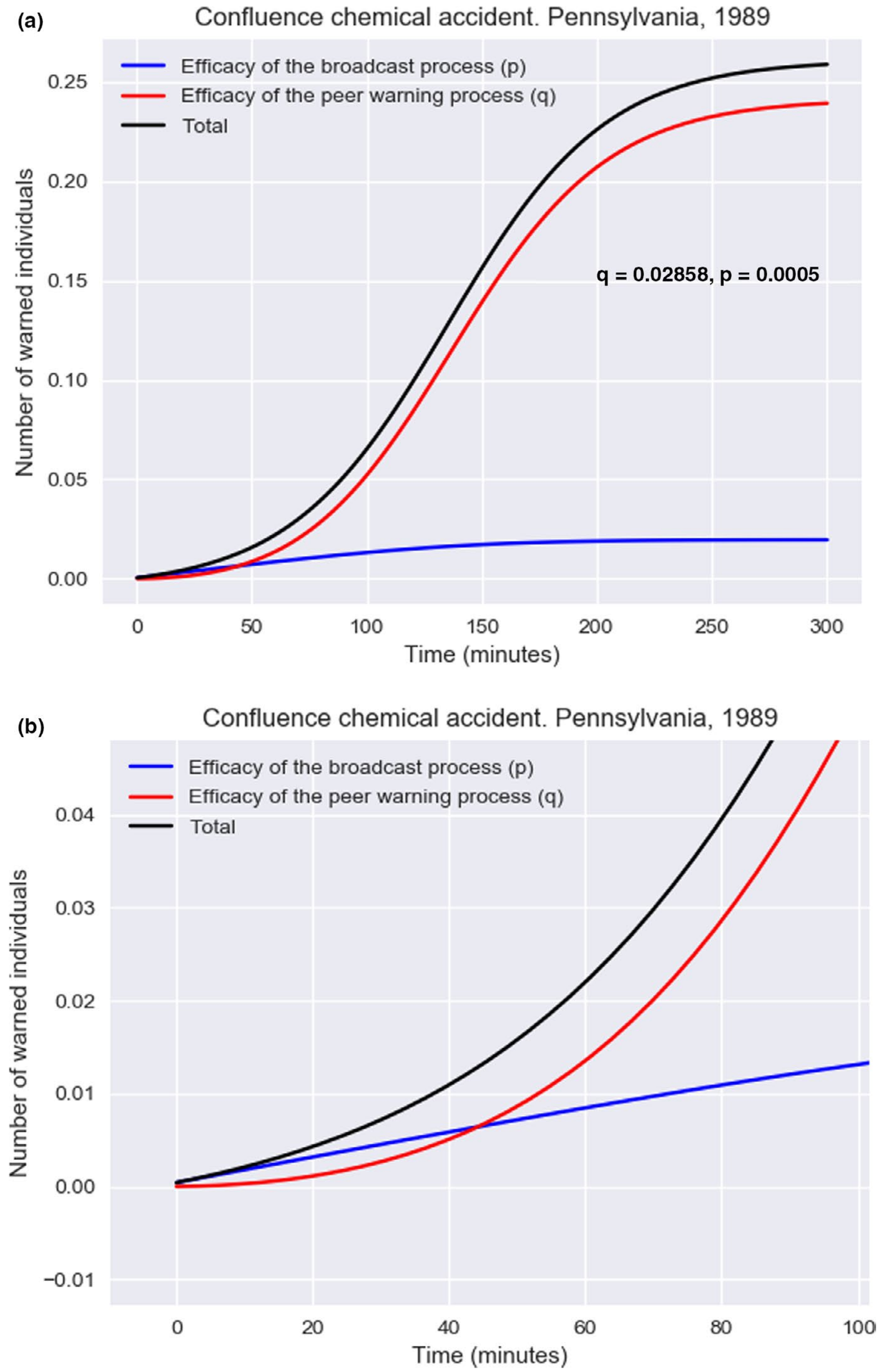


FIGURE 4 Application of the Bass model to the Confluence precautionary evacuation. (a) The cumulative number of warned individual is represented, divided into three categories: Black line = total number of warned individuals. Blue line = Number of individuals warned by the broadcast process (related to *q*) and Red line = Number of individuals warned by the peer-to-peer process (related to *p*). (b) Detail of in the time slot 0–100 min [Colour figure can be viewed at wileyonlinelibrary.com]

Sorensen, 1989). Vertical media are represented by radio, television, governments, news and reporters, while horizontal warning networks are represented by neighbours, kin (relatives) and other social ties (friends and coworkers). However, the development of a comprehensive model is challenging since the two components may present significant variations related to the specific emergency, to the social structure and to the cultural environments in which communication processes are immersed. Another difficulty is that the contagion warning network's elements are numerous and can only be modelled as an aggregate and complex rather than identified individually (Lindell et al., 2007)).

Social networks play a major role in emergency situations and shape the response to emergency warnings. An extensive review of the use of social media in emergency situations is provided by Simon (Simon, Goldberg, & Adini, 2015) and Reuters (Reuter & Kaufhold, 2017). Recent research reported that ignoring the contribution of the informal warning network can cause a systematic downward bias in the warning reception time distribution, thus overestimating the amount of time needed to warn a risk area population (Lindell et al., 2007). However, the impact of the social network on the response to an emergency situation may even depend on social and political factors. For instance, in a recent paper Martin-Shields studied the diffusion of information during a crisis in the Independent State of Samoa. He concluded that the political and social context plays a major role and that the majority of respondents in Samoa preferred vertically integrated communications systems when it came to making decisions during a crisis, that is Samoa citizens rely heavily on broadcast media and official channels for information in disaster response (Martin-Shields, 2019).

Building on previous studies (Rogers & Sorensen, 1991; Lindell & Perry, 1987; Lindell et al., 2007; Rogers & Sorensen, 1989), the purpose of this paper was to present the results of an analysis on the timing of diffusion of the warning messages. In this paper, the Bass model is introduced to study the spread of information during emergency situations. Survey data collected in two accidents involving hazardous materials are used to validate the Bass model and to assess the time evolution of emergency warning messages.

A number of interesting conclusions can be drawn. First off, the Bass model provided an excellent fit of the warning diffusion times related to the Pittsburgh phosphorus oxychloride release. With reference to Figure 2b, the application of the Bass model suggests that the first phase of the warning process is sustained by a broadcast information diffusion process (related to p coefficient. Namely, $p = .0011$). The broadcast component disseminates the emergency warning in a centralized way, that is many are alerted simultaneously. However, after about 35 min from the beginning of the warning process (see Figure 2b), the efficacy of the warning process itself is dominated by the "imitation" component, that is the effectiveness of a robust social network between individuals ($q = 0.038$). In fact, Figure 2b pinpoints that after about 35 min from the occurrence of the accident, the number of warned individuals alerted by the peer-warning process exceeds the number of people warned by broadcast media. Interestingly, our findings are confirmed by the original work of Rogers and Sorensen (Rogers & Sorensen, 1989), stating that: "*The most effective warning source in terms of average time to warn in Pittsburgh was the contagion of the warning message through the social network*."

A similar analysis was performed for the precautionary evacuation occurred in Confluence. With reference to Figure 4b, the application of the Bass model suggests that the first phase of the warning process is prompted by a broadcast information diffusion process ($p = .0005$). By analogy with the chemical release in Pittsburgh, in the Confluence accident, after about 45 min from the beginning of the warning process (see Figure 4b), the efficacy of the warning process itself is dominated by the "imitation" component ($q = 0.02858$). Furthermore, our results are consistent with the original formulation of the Bass model providing q coefficients at least an order of magnitude greater than p, proving that social communication has a greater effect on the diffusion of information than broadcast media (Bass, 1969).

As a general rule, our results indicate that accurate control of information may be essential within approximately 40–60 min from the beginning of the emergency event. After such temporal frame, the presence of a social network between individuals is likely to dominate the information diffusion process. Interestingly, our findings correlate favourably with previous results. A recent paper by Del Vicario and colleagues (Del Vicario et al., 2016) studied the spreading of information among social network users analysing the diffusion of conspiracy theories and scientific information. They showed that the online dissemination of information exhibits a probability peak in the first 2 hr and a second after about 20 hr. In the same paper, they found that a significant percentage of the information diffuses rapidly (24.42% of the science news and 20.76% of the conspiracy rumours diffuse in less than 2 hr, and 39.45% of science news and 40.78% of conspiracy theories in less than 5 hr).

The Bass model presented in this paper can be successfully applied in disaster responses that allow for the presence of both warning diffusion components (i.e. broadcast and contagion). A striking feature of the Bass model is its possible application in some special emergency situations where $p = 0$ or $q = 0$. As shown by Equation (2), the broadcast process is described by an exponential growth while the contagion process yields a logistic curve. As a matter of fact, one would expect that the broadcast component is likely to play a major role in situations of emergency where ample forewarning can given to the population (i.e. hurricanes) (Lindell & Perry, 1992). Indeed, it is likely that people are already in high state of alert and are awaiting for warning messages from media, governments of local authorities; therefore, vertical sources of information are more effective than horizontal ones. Recent research reports that in such circumstances, it is expected that the diffusion of warning messages can be approximated by the following function (Lindell & Perry, 1992): $N_t = 1 - e^{(-at^b)}$, where N_t is the fraction of people warned at time t, with a and b being two parameters derived from the fit. Consistently, if $q = 0$ is considered in the Bass model described by Equation (2) a similar function is obtained, that is $N_t = 1 - e^{(-at)}$ thus proving the possible extension of the model to special cases where the contagion component (i.e. p) is missing. Further studies will need to be undertaken to explore the possibility of extending the Bass model to other emergency situations.

6 | CONCLUSIONS

Understanding the diffusion of warning messages is essential to adequately respond to emergency events and situations. According to the present work, the general Bass model applied to the time evolution of emergency warnings fits literature data very well and has the potential to provide key information that can be applied right away by professional communicators, advisors and decision-makers in case of a CBRNe event requiring urgent warning diffusion. In particular, accurate management of information spreading may be essential within approximately 40–60 min from the beginning of the emergency event. After such temporal frame, social networks (i.e. the "imitation" component of the Bass model) are likely to dominate the information diffusion process. This finding concurs well with previous findings (Del Vicario et al., 2016; Lindell et al., 2007). Ultimately, the control of information and emergency warnings at the very beginning of a disaster or of an emergency event is key to counter the generation of misinformation, rumour and fake information.

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CONFLICT OF INTEREST

The authors have no conflicts of interest to declare.

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